**%Question 1**

**Part a:**

function LAB06ex1

clc

omega0 = 2; c = 1; omega = 1.4;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



computed amplitude of forced oscillation = 0.40417

theoretical amplitude = 0.40417

T=2pi/w=2pi/1.4=4.49 seconds

alpha=arctan(cw/(w0squared-wsquared))

alpha=atan(c\*omega/(omega0^2-omega^2)) = .6015 radians

**Part b:**

function LAB06ex1

clc

omega0 = 2; c = 1; omega = 1.4;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

alpha=atan(c\*omega/(omega0^2-omega^2));

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

figure (2)

yc = y-C\*cos(omega\*t-alpha)

plot (t,yc); grid on

title ('complementary solution')

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



The oscillation appears to be decreasing exponentially because the amplitude is decreasing at a steady pace until the graph ultimately becomes a straight line at x=0.

**%Question 2**

**Part a:**

function LAB06ex2

omega0 = 2; c = 1;

OMEGA = 1:0.02:3;

C = zeros(size(OMEGA));

Ctheory = zeros(size(OMEGA));

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50; t1 = 25;

for k = 1:length(OMEGA)

omega = OMEGA(k);

param = [omega0,c,omega];

[t,Y] = ode45(@f,[t0,tf],Y0,[],param);

i = find(t>t1);

C(k) = (max(Y(i,1))-min(Y(i,1)))/2;

Ctheory(k) = 1/sqrt((omega0^2-omega^2)^2+c^2\*omega^2);

end

figure(2)

plot(OMEGA,C,'b',OMEGA,Ctheory,'ro-'); grid on; %

xlabel('\omega'); ylabel('C');

%---------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



**Part b:**

w = 1.87, Cmax = .517

**Part c:**

symsW;

f(W)= 1/sqrt((omega0^2-W^2)^2+(c\*W)^2);

y=diff(f);

z=solve(y)

f(z)

Result in command window:

z =

0

14^(1/2)/2

-14^(1/2)/2

ans =

1/4

(2\*15^(1/2))/15

(2\*15^(1/2))/15

This analytical value of omega is slightly less (1.871) than the approximation made in part b.

**Part d:**

function LAB06ex1

clc

omega0 = 2; c = 1; omega = 1.87;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

alpha=atan(c\*omega/(omega0^2-omega^2));

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



Computed amplitude of forced oscillation = 0.5164. Theoretical amplitude = 0.5164. The newly computed amplitude of forced oscillation using the value of omega from part C is greater than the computed amplitude of forced oscillations in question one. The amplitude of forced oscillations will be less than 0.5 when omega is less than or greater than the resonant frequency omega-naught.

**Part e:**

function LAB06ex2

omega0 = 2; c = 1;

OMEGA = 1:0.02:3;

C = zeros(size(OMEGA));

Ctheory = zeros(size(OMEGA));

t0 = 0; y0 = 12; v0 = 15; Y0 = [y0;v0]; tf = 50; t1 = 25;

for k = 1:length(OMEGA)

omega = OMEGA(k);

param = [omega0,c,omega];

[t,Y] = ode45(@f,[t0,tf],Y0,[],param);

i = find(t>t1);

C(k) = (max(Y(i,1))-min(Y(i,1)))/2;

Ctheory(k) = 1/sqrt((omega0^2-omega^2)^2+c^2\*omega^2);

end

figure(1)

plot(OMEGA,C,'b',OMEGA,Ctheory,'ro-'); grid on; %

xlabel('\omega'); ylabel('C');

%---------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



The results are not affected by the changes made to the initial conditions. The expression for C does not depend on the initial conditions.

**%Question 3**

**Part a:**

function LAB06ex2

omega0 = 2; c = 0;

OMEGA = 1:0.02:3;

C = zeros(size(OMEGA));

Ctheory = zeros(size(OMEGA));

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50; t1 = 25;

for k = 1:length(OMEGA)

omega = OMEGA(k);

param = [omega0,c,omega];

[t,Y] = ode45(@f,[t0,tf],Y0,[],param);

i = find(t>t1);

C(k) = (max(Y(i,1))-min(Y(i,1)))/2;

Ctheory(k) = 1/sqrt((omega0^2-omega^2)^2+c^2\*omega^2);

end

figure(1)

plot(OMEGA,C,'b',OMEGA,Ctheory,'ro-'); grid on; %

xlabel('\omega'); ylabel('C');

%---------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



There is a sharp peak at the resonant frequency value at omega-naught-2, the solution skyrockets until it equals 12.1. Maximal amplitude = 12.1. The frequency yielding the maximal amplitude of the forced solution is 2. Compared to omega0, they are both the same.

**Part b:**

function LAB06ex1

clc

omega0 = 2; c = 0; omega = 2;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

alpha=atan(c\*omega/(omega0^2-omega^2));

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



The amplitude of the graph increases. This graph illustrates a vibration at resonance.

**%Question 4**

**Part a:**

function LAB06ex1

clc

omega0 = 2; c = 0; omega = 1.8;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 100;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

alpha=atan(c\*omega/(omega0^2-omega^2));

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

g(t,omega0,omega);

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];

function envelope = g(t,omega0,omega)

C = 1/(abs(omega0^2-omega^2));

A = 2\*C\*sin(.5\*(omega0-omega)\*t);

plot(t,A,'r-',t,-A,'g-')



function LAB06ex1

clc

omega0 = 2; c = 0; omega = 1.8;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 100;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

alpha=atan(c\*omega/(omega0^2-omega^2));

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

holdon

C = 1/(abs(omega0^2-omega^2)); A = 2\*C\*sin(.5\*omega0\*t-0.5\*omega\*t);

plot (t,A,'r')

plot (t,-A,'g')

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



**Part b:**

The period of the fast oscillation = 3.3



**Part c:**

Length=31.41

**Part d:**

function LAB06ex1

clc

omega0 = 2; c = 0; omega = 1.9;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 100;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

alpha=atan(c\*omega/(omega0^2-omega^2));

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];

****

T=3.22 Length=62.83

The period of the fast oscillations decreased and the length of the beats increased.

function LAB06ex1

clc

omega0 = 2; c = 0; omega = 1.6;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 100;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];

****

T=3.49 Length=15.07

The period of rapid oscillations increased and the length of the beats decreased.

**Part e:**

function LAB06ex1

clc

omega0 = 2; c = 0; omega = 0.5;

param = [omega0,c,omega];

t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 100;

options = odeset('AbsTol',1e-10,'relTol',1e-10);

[t,Y] = ode45(@f,[t0,tf],Y0,options,param);

y = Y(:,1); v = Y(:,2);

figure(1)

plot(t,y,'b-'); ylabel('y'); grid on;

%hold on;

t1 = 25; i = find(t>t1);

C = (max(Y(i,1))-min(Y(i,1)))/2;

disp(['computed amplitude of forced oscillation = ' num2str(C)]);

Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c\*omega)^2);

disp(['theoretical amplitude = ' num2str(Ctheory)]);

%----------------------------------------------------------------

functiondYdt = f(t,Y,param)

y = Y(1); v = Y(2);

omega0 = param(1); c = param(2); omega = param(3);

dYdt = [ v ; cos(omega\*t)-omega0^2\*y-c\*v ];



There are no beats present. This is so because there is no overlapping in the solution when omega equals 0.5 or less. The initial condition and the current frequency are too far apart for overlapping to occur.